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Nonlinear Dynamics of a Thin Plate in a Nonstationary Electromagnetic Field of the Inductor

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Abstract
The problems of the non-stationary deformation of a thin round plates in the electromagnetic field (EM-field) created by a massive inductor are considered. The finite element method is used for the analysis of spatial-temporal distribution of the electromagnetic field components and the parameters of the stress-strain state in the system of the «inductor-billet (plate)» with a consideration of the air layer. The data of measurements in experimental studies of vibrations of plates in pulsed EM-field are used to establish the frequency modes with the maximum values ponderomotive forces generated by an EM-field. The solution of the problem of plate vibrations was obtained. The temporal and spatial distribution of components of the electromagnetic field, displacements and stresses on the surface of the plate with the analytical solutions and experimental data are compared. Areas with the maximum displacements and stresses in the plate were installed for some «configuration» of electromagnetic field.

Keywords
Electromagnetic field, vibrations, deforming

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Introduction

The interaction of electro-conductive bodies with the electromagnetic field (EM-field) leads to occurrence in them of the solenoidal currents, which in turn causes the appearance of ponderomotive forces leading to movement and deformation of the conductive bodies. Influence of intensive EM-fields is exposed to the elements of various technical devices: various converters of energy (generators, transformers), devices for the geological exploration with the help of pulsed EM-field, the system for processing of materials by EM-field forces. The energy created in the EM-field, can reach such values, in which the strength of structural elements is violated. In addition, the movement of structural elements in an EM-field (vibrations) can lead to the distortion of the distribution of EM-field components, which might disturb the normal mode of work of the device.

Thus, the problem of the analysis of component of the EM-field and the subsequent analysis of the stress-strain state (SSS) is relevant in the scientific and practical terms. It should be noted that a large number of scientists in different times focus special attention to problems of the analysis of the thermo-mechanics in presence of the EM-field. A significant contribution was made by S.A. Ambratcumyan, Ya.Yo. Burak, O.S. Wol’mir, A.R. Gachkevich, O.M. Guz’, S.A. Kaloerov, Ya. S. Pidstryhach etc. However, basically these researches are focused on the analysis of elastic behavior of material, and the solutions are given for bodies of the canonical form. Thus, the problem for the creation of effective methods of calculation of magneto-elastic plasticity for a body of arbitrary shape in the present time is actual one.

The paper presents the analysis of the spatial-temporal distribution component of the EM-field in the system for electromagnetic forming (EMF) of materials and the subsequent analysis of deformation of the elements of this system. EMF is one of the progressive methods of processing of materials, based on the ability of metals to deform plastically under the action of ponderomotive forces. In this case, the high levels of ponderomotive forces on the one hand positively influence on process of plastic deformation of the workpiece, on the other hand, they negatively affect the resistance of the inductor systems, because it may violate the strength of the inductor.

1. Problem Statement

Let's consider one variant of the inductors designed a deformation of thin-walled billets. In this case, it is a massive body of rotation of the complex form (fig.1). In the center of the inductor the bore (window) was made in the form of a truncated cone. It is necessary to analyze the distribution of the component of the EM-field in the system of the «inductor-billet», consider the vibration process in the system, to assess the SSS of the system.

Figure 1. The inductor with cone bore

The mathematical formulation of the problem in this case has the following form. The equations of motion:

$$\sigma_{ij,j} + F_{pi} = \rho \ddot{u}_1 ,\ \vec{F}_p(x_i) = \rho \vec{E} + \mu_c \left[\vec{j} \times \vec{H} \right] \tag{1}$$

where u_i are components of the vector of displacements, σ_{ij} are components of the stress tensor, ρ is the mass density, F_{pi} are components of the vector of volumetric forces of Lorentz, $\vec{j}$ is the vector of current density , $\vec{E}$, $\vec{H}$ are vectors of the intensity of electric and magnetic fields, μ_c is the magnetic permeability. The electromagnetic field is determined by the system of the Maxwell's equations [1]:

$$rot\ \vec{H} = \varepsilon_c \frac{\partial \vec{E}}{\partial t} + \rho\omega\dot{\vec{u}} + \vec{j} ,\ rot\ \vec{E} = -\mu_c \frac{\partial \vec{H}}{\partial t} ,\ div\vec{H} = 0 ,\ \varepsilon_c div\vec{E} = \rho\omega \tag{3}$$

where ω is the density of electrical charges, ε_c is the electrical permeability. Equations (1) and (3) are supplemented by physical relations:

$$\vec{D} = \varepsilon_c \vec{E} ,\ \vec{B} = \mu_c \vec{H} ,\ \vec{j} = \gamma_c \vec{E} + \gamma_c \left[\dot{\vec{u}} \times \vec{B} \right],\ \varepsilon_{ij} = A^e_{ijkl} \sigma_{kl} \tag{4}$$

where $\vec{D}$, $\vec{B}$ are induction vectors of electric and magnetic fields, γ_c is an electrical conductivity of the material, ε_{ij} are the components of a tensor of strain, A^e_{ijkl} is the components of a tensor adopted for the description of the properties of the material, within the limits of linear elasticity of the material, the ratio meet the generalized Hook's law. For isotropic body the material constant tensor is defined as: $A^e_{ijkl} = \frac{1}{E}\left[(1+\nu)\delta_{ik}\delta_{jl} - \nu\delta_{ij}\delta_{kl} \right]$, where E, ν are the elastic modulus and Poisson's ratio. The relationship between deformations and displacements will be considered in the framework of the linear Cauchy relations:

$$\varepsilon_{ij} = 1/2\left(u_{j,i} + u_{i,j}\right) \tag{5}$$

The problem is supplemented by boundary conditions:

$$\vec{E}_{\Gamma} \times \vec{n} = 0, \quad \vec{D}_{\Gamma} \cdot \vec{n} = 0, \quad \vec{H}_{\Gamma} \times \vec{n} = 0, \quad \vec{B}_{\Gamma} \cdot \vec{n} = 0; \quad \vec{\sigma}_n = \vec{p}_n + \frac{\Xi}{2}\vec{E}_{\Gamma} + \frac{\mu_c}{2}\left(\Xi\dot{\vec{u}}_{\tau} + \vec{i}\right)\times\vec{H}_{\Gamma} \tag{6}$$

where $\vec{\sigma}_n = \sigma \cdot \vec{n}$ is the vector of mechanical stresses on the boundary with the normal $\vec{n}$, $\Xi, \vec{i}$ – density of surface charges and currents, $\dot{\vec{u}}_{\tau}$ are the projection of the velocity vector of a point on a plane tangent to the boundary of the body.

2. Solution and Analysis of the Results

2.1 The analysis of the distribution component of the EM-field in the system of the «inductor-billet»

Let us consider the distribution of components of the EM-field in the system of the «inductor-billet» with the help of the finite element method (FEM). In the first approximation inductor can be modeled as an axisymmetric body [2]. The calculation scheme of the problem of determination of space-time configuration of the EM-field is shown in figure 2. In this case, the system of Maxwell's equations in a cylindrical coordinate system relative to the component of the magnetic field intensity, in the mind of axial symmetry, is reduced to the form (8):

$$\begin{cases} rot\vec{H}(r,z,t) = \dfrac{\partial H_z}{\partial r} - \dfrac{\partial H_r}{\partial z} = 0; \\ div\vec{H}(r,z,t) = \dfrac{1}{r}\cdot\dfrac{\partial}{\partial r}\left(r \cdot H_r\right) + \dfrac{\partial H_z}{\partial z} = 0. \end{cases} \tag{8}$$

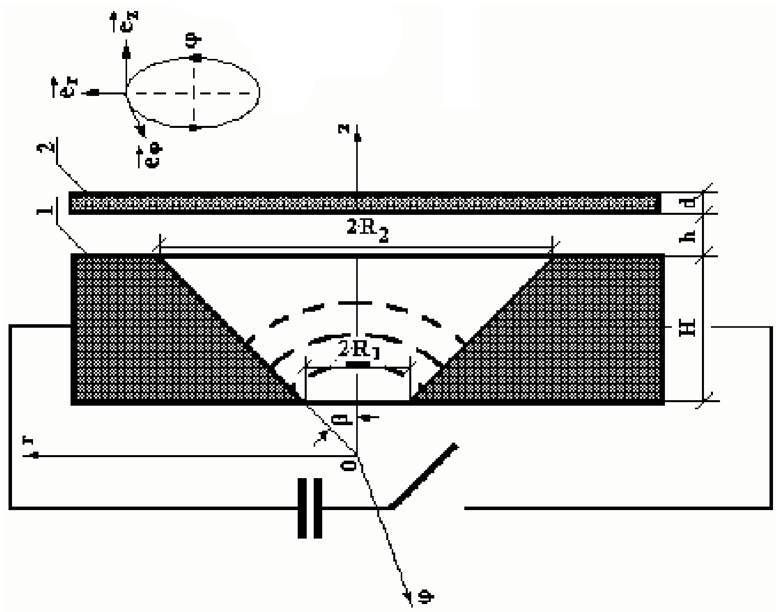

Figure 2. Calculation model of the analysis of the EMF in system of the «inductor-billet»

To solve this problem use a FEM that allows to largely avoiding the various simplifying assumptions. In addition, it is necessary to have in mind that the transfer of the current of the inductor for billet takes place through the intermediate environment – air, which is excluded from consideration in the analytical solution. Enter into a consideration the magnetic vector potential [3] – $\vec{A}(r,z,\varphi,t)$: $\vec{B} = rot\vec{A}$. In this case, the magnetic vector potential has only one non-zero component $A_{\varphi} = A$. Compare the obtained solution with the analytical and experimental data [2]. Figure 3 shows the distribution of the radial components of the intensity of electromagnetic field on the surface of the billet directly in the neighborhood of the window field. The solid line is the solution obtained FEM, dashed line – analytical solution, points – experimental data [3]. From the figure it is shown a good agreement between the numerical and an analytical solution, which testifies to expediency of application of the FEM for the analysis of the distribution of EM-field in the systems similar to, considered one.

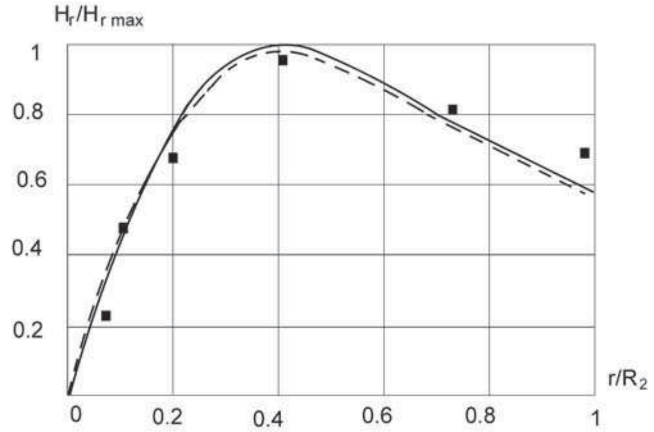

Figure 3. The comparison of the distribution of the radial component of magnetic field intensity on the surface of the billet

2.2 Analysis of vibrations of a round plate in a variable magnetic field

The variability in time of the EM-field components, obviously, leads to vibrations of elements of the «inductor-billet». Due to the fact, that the inductor is much more massive than the workpiece, the vibrations of the workpiece are most pronounced. For their modeling consider vibrations of a thin round plate in a variable magnetic field, which is a transverse to the surface of the plate – fig.4. In this case, for the analysis of bending vibrations of plates we can use the equation of the [4]:

$$\begin{aligned} & D\nabla^4 w + 2\rho h\ddot{w} - h\frac{\partial}{\partial x}\left(\sigma_{zx}^{+} + \sigma_{zx}^{-}\right) - h\frac{\partial}{\partial y}\left(\sigma_{zy}^{+} + \sigma_{zy}^{-}\right) - \left(\sigma_{zz}^{+} + \sigma_{zz}^{-}\right) = \\ & = \frac{\partial}{\partial x}\int_{-h}^{h} f_x z dz + \frac{\partial}{\partial y}\int_{-h}^{h} f_y z dz + \int_{-h}^{h} f_z dz \end{aligned} \tag{9}$$

where $\sigma_{zx}^{+}, \sigma_{zx}^{-}, \sigma_{zy}^{+}, \sigma_{zy}^{-}, \sigma_{zz}^{+}, \sigma_{zz}^{-}$ – the stresses at the top and bottom of the outer boundaries of the plates, f_x, f_y, f_z – components of the Lorentz force, $D = \dfrac{2Eh^3}{3\left(1-\nu^2\right)}$ – the bending stiffness of the plate.

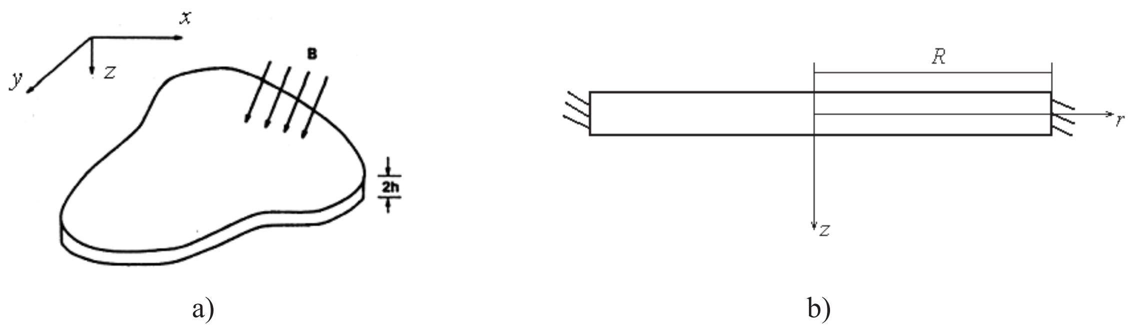

Figure 4. Calculation model of the analysis of vibrations of a round plate in a variable magnetic field

We approximated the distribution of the functions of the bending of the plate along the radial coordinate of the function:

$$w = A(t)\left(R^2 - r^2\right)^2 \tag{10}$$

where $A(t)$ – the value of the bending in the center of the plate, t – the time. This function automatically satisfies the boundary conditions type of rigid fixation. Let us pass to dimensionless designations of variables and functions: $\bar{r} = \frac{r}{R}$; $\bar{A} = \frac{A}{h}$; $\bar{w} = \frac{w}{h}$. Then, taking into account dependencies (4) and (5), and taking into account the axial symmetry of the equation (9) is converted to the form:

$$D\nabla^4 \bar{w} + \frac{2h}{\mu_0}\left(1 - \frac{\mu_0}{\mu}\right) B^2 \nabla^2 \bar{w} + 2\rho h \ddot{\bar{w}} = f_z + f_r \tag{11}$$

After substitution, we obtain:

$$64D\bar{A}R^4 + \frac{16h}{\mu_0}\left(1 - \frac{\mu_0}{\mu}\right) B^2 \bar{A} R^4 \left(2\bar{r}^2 - 1\right) + 2\rho h \ddot{\bar{A}} R^4 \left(1 - \bar{r}^2\right)^2 = f_z + f_r \tag{12}$$

The components of the Lorentz force can be presented according to [5] in the form of:

$$f_z + f_r = e^{-\delta_0 \omega t} \cos(\omega t) \left[\frac{2 + \bar{r}^2}{1 + \bar{r}^2}\right], \tag{13}$$

where ω is a frequency of current in pulse. Let us consider the solution of the tasks for the following data:

$h = 0{,}0005(\text{ì})$, $R = 0.07(\text{m})$, $\delta_0 = 0{,}25$, $B = 40$, $\mu = 2$, $\mu_0 = 1{,}257 \cdot 10^{-6}$, $\omega = 17{,}9 \cdot 10^3 \left(\text{s}^{-1}\right)$,

pulse duration – of 0.001(s). Fig. 5 shows the plot of the time-dependence of the bending of a centre of a plate. The graph shows that the pulse of the impact of the fixed plate makes damped oscillations, which are continuing in time longer than the current pulse. Also note that the first maximum displacement, attributable to the initial moment of time, is almost twice more than the second maximum. It is logical to assume that this is the first maximum corresponds to the occurrence of the maximum levels of stress, which can lead to the appearance of plastic deformations. Thus, in the analysis of deformation of systems for EMF, the task can be considered in the quasi-stationary set, which corresponds to, in fact, a review of the distribution of EM-field in the initial moment of time.

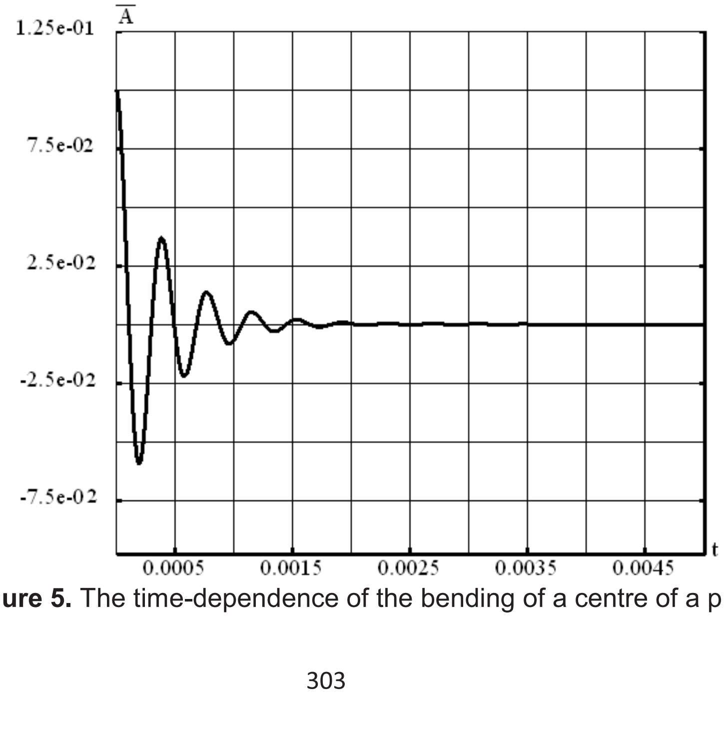

Figure 5. The time-dependence of the bending of a centre of a plate

2.3 The analysis of the SSS of the system «inductor-billet» in the case of quasi-stationary EM-field

The analysis of the SSS of the system «inductor-billet» was made with the help of FEM, means the software package (SP) ANSYS. Baseline data for the analysis of the SSS served as the distribution component of the EM-field. As is known, the ponderomotive forces can lead both to the repulsion and to the attraction of conducting bodies. It is experimentally shown [2] that the low-frequency current in pulse inductor mechanisms for attraction prevails over the mechanisms of repulsion. An ideal variant, in which the attraction is the impact of a constant current of great strength, but in reality to implement such a process, is not possible. At the same time calculated in the approximation of quasi-stationary process provides an opportunity to carry out qualitative assessments of the deformation of the elements of the system. It is in this setting, and the calculation was done. The picture of the deformed state of the system «inductor-billet» is presented in Fig. 6. Note that the most intensive process of deformation is directly opposite the cone of the window field. Also as results of the analysis of the SSS have been identified zones, in which the stress intensity reaches maximum values and it is in these areas should be expected of plastic deformations.

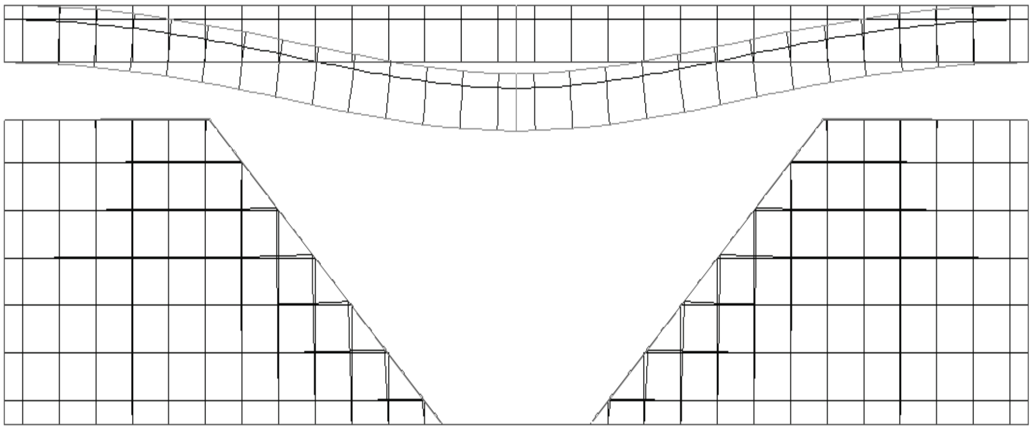

Figure 6. The deformed shape of the system «inductor-billet»

Conclusions

In the article the problems of the analysis of component of the EM-field for a system of interacting bodies and the subsequent analysis of deformation have been shown. The calculations on the example of the system of the «inductor-billet», which are used in the processes of EMF, have been made. The analysis of the distribution component of the EM-field and the subsequent analysis of deformation produced by FEM. Elements of the system «inductor-billet» are considered in the framework of the united calculation model. Vibrations of a thin conducting of the round plate in a variable magnetic field have been analyzed.

References

[1] Maxwell J.C. *A treatise on Electricity and Magnetism,* Clarendon Press, Oxford, 1873.

[2] Turenko A.N., Batygin Yu.V. and Gnatov A.V. *Pulse magnetic fields to progressive technologies*: *Theory and experiment of thin-walled metal attraction by pulsed magnetic field,* KhNADU, Kharkov, 2009 (in Russian).

[3] Tozoni O.V., Mayergoiz I.D. *Calculation of three-dimensional electromagnetic fields,* «Tekhnika», Moscow, 1974 (in Russian).

[4] Ambratcumyan S.A., Baghdasaryan G.E. and Belubekyan M.V. *Magneto elastic of thin shells and plates,* Nauka, Moscow, 1977 (in Russian).

[5] Batygin Yu.V., Gnatov A.V. and Shigoleva S.A. The direction of the forces acting on the sheet ferromagnetic, depending on the temporal characteristics of the PMFM, *Electrotehnika ta elektromechanika*, Vol. 3., pp.56-61, 2011 (in Russian).